PREVIOUS QUESTIONS AND ANSWERS

MATHS(311)

1. In a small scale industry, a manufacturer produces two types of book cases. The first type of book case requires 3 hours on machine A and 2 hours on machine B for completion whereas the second type of book case requires 3 hours on machine A and 3 hours on machine B. The machines A and B respectively run for at the most 18 hours and 14 hours per day. He earns a profit of 30 on each type of case of first type and 40 on each book case of second type. How many book cases of each type should he manufacture so as to have maximum profit. Make it an LPP and solve it graphically. (2019)

Machine	Processing Time Of The Products		Available Time
	Type I (x units)	Type II (y units)	
A B	3 hrs 2 hrs	3 hrs 3 hrs	18 hrs 14 hrs
Profit per unit	Rs.30	Rs.40	

- **Tabulation of details – 1mark**
- **Constraints and objective function – 1 ½ marks**

  $x + y \leq 6$, $2x + 3y \leq 14$, $x \geq 0$, $y \geq 0$, $z = 30x + 40y$
- **Graph – 1 ½ marks**

  A (0,6), B (0,6), C (0,4.7), D (7,0)
- **Optimum solution – 1 mark**

  Feasible region – O (0,0), B (6,0) C (0,4.7), E (4,2)
- **Maximum profit – 1 mark**

  Max Profit = 200/-, Type I – 4 units and Type II – 2 units

2. Prove that the lines

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \text{ and } \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$$ are coplanar. Find the equation of plane containing these lines. (2024)

$$\begin{vmatrix} 3 & 7 & 11 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0 - 1\,mark$$

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- **3 (35 – 28) – 7 (21 – 7) + (12 – 5) = 0 – 1 mark**
- **0 = 0 ; coplanar - 1 mark**

$$\begin{vmatrix} x+1 & y+3 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0 - 1\,mark$$

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- **(x + 1)(35 – 28) – (y + 3)(21 – 7) + (z + 5)(12 – 5) = 0 – 1 mark**
- **x – 2y + z = 0 – 1 mark**

3. Using matrices solve the system of equations. (2016)
   2x – y + z = 3, -x + 2y – z = -4, x – y + 2z = 1

- **A =** $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ $\quad \mathbf{B} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$

  AX = B ; X = $A^{-1}\mathbf{B} - \frac{1}{2}$ **mark**
- **|A| = 4 – ½ mark**
- **Adj A =** $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} - 2\,mark$
- $A^{-1} = \frac{1}{|A|} Adj\, A = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} - 1\,mark$
- $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} - \frac{1}{2} mark$
- $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ -8 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} - 1\,mark$
- **x = 1, y = -2, z = -1 – ½ mark**

4. Evaluate $li\,m_{x\to 1}\frac{x^3-1}{x-1}$ (2018)

- $\frac{x^3-1^3}{x-1} = \frac{[(x-1)(x^2+1^2+x)]}{x-1} - 1\,mark$
- $li\,m_{x\to 1}\, x^2 + x + 1 = 1^2 + 1 + 1 = 3 - 1\,mark$

5. Find dy/dx, if y = $e^{x^2} + 2\sin x - \frac{5}{3}e^x + 2e$ (2018)

- $\frac{dy}{dx} = \frac{d}{dx}e^{x^2} + \frac{d}{dx}2\sin x - \frac{d}{dx}\frac{5}{3}e^x + \frac{d}{dx}2e - 1\,mark$
- $e^{x^2}\frac{d}{dx}x^2 + 2\frac{d}{dx}\sin x - \frac{5}{3}\frac{d}{dx}e^x + 0 - 1\frac{1}{2}\,marks$
- $e^{x^2}\,2x + 2\cos x - \frac{5}{3}e^x - 1\frac{1}{2}marks$

6. Write the converse of the following statements (2019)
    a) If game is cancelled, then team A is win.
    b) If a is a multiple of b then b is a factor of a.

- **a)If team A is win, then game is cancelled – 1 mark**
- **b)If b is a factor of a then a is a multiple of b – 1 mark**

7. Find the derivative of sin x^3 with respect to x (2019)

- $$\frac{d}{dx}\sin x^3 = \cos x^3 \frac{d}{dx}x^3 - 1\ mark$$
- $$= \cos x^3 . 3x^2 - 1\ mark$$

8. Find the inverse of the matrix using elementary operations $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (2021)

- **A = IA ;** $$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A - \frac{1}{2}\ mark$$
- **R_1** $$\rightarrow R_1 + R_2 \begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A - \frac{1}{2}\text{mark}$$
- $$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - 3R_1 \begin{bmatrix} 1 & 3 & 5 \\ 0 & -1 & -2 \\ 0 & -8 & -14 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ -3 & -3 & 1 \end{bmatrix} A - 1\text{ mark}$$
- $$R_2 \rightarrow - R_2 \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & -8 & -14 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ -3 & -3 & 1 \end{bmatrix} A - \frac{1}{2}\text{mark}$$
- $$R_1 \rightarrow R_1 - 3R_2, R_3 \rightarrow R_3 + 8R_2 \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A - 1\ mark$$
- $$R_3 - \frac{R_3}{2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A - \frac{1}{2}\text{mark}$$
- $$R_1 \rightarrow R_1 + R_3, R_2 \rightarrow R_2 - 2R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A - 1\ mark$$

- $$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A - 1\, mark$$

9. Show that the relation R = {(a,b):a>b} on N is transitive but not symmetric. (2023)

- **(x,y) element of R** $\rightarrow \mathbf{x > y \rightarrow y < x \rightarrow (y, x)}$ **not element of R.** **Therefore not symmetric – 1 mark**
- $\mathbf{(x, y), (y, z)}$ **element of R** $\mathbf{\rightarrow x > y, y > z \rightarrow x > y > z \rightarrow x > z \rightarrow (x, z)}$ **element of R.** **Therefore transitive – 1 mark**

10. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ (2020)

- **LHS - Let √x = tan** $\boldsymbol{\theta\,;\ \tan^{-1}\sqrt{x} = \theta -}$ **1 mark**
- **RHS –** $$\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}\times 2\theta = \theta - 1\ \text{mark}$$

11. Find the rate of change of area of a circle with respect to it’s variable radius r, when r = 3 cm. (2016)

- **Area of the circle = πr² - ½ mark**
- **Rate of change of area =** $\frac{d}{dr}(\pi r^2) = 2\pi r - 1\, mark$
- **When r = 3 cm, dA/dr = 2π × 3 = 6π – ½ mark**

12. Let * be a binary operation defined by a*b = b – 2a , then the value of (1*2)* 3 is _____. (2024)

- **1*2 = 0, 0*3 = 3 – 1 mark**

13. A relation R on any set A is said to be _____, if (a, b) element of R ,(b, c) element of R implies (a, c) element of R for all a, b, c element of A. (2024)

- **Transitive – 1 mark**

14. Combine the following statements using “if and only if” : (2022)
    p : If a parallelogram is a rhombus, then all its four sides are equal.
    q : If four sides of a parallelogram are equal then the parallelogram is a rhombus.

- **A parallelogram is a rhombus if and only if its four sides are equal – 2 mark**

15. Which of the following statement is true? (2021)

(A) Chord of a circle is double of its radius

(B) Concentric circles have different radius

(C) If a number has more than two factors then it is not composite

(D) 25 is a multiple of 8

- **Option B – 1 mark**